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Dynamical couplings, dynamical vacuum energy and confinement/deconfinement from R^2-gravity

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ABSTRACT

We study within Palatini formalism an $f(R)$-gravity with $f(R) = R + \alpha R^2$ interacting with a dilaton and a special kind of nonlinear gauge field system containing a square-root of the standard Maxwell term, which is known to produce confinement in flat space–time. Reformulating the model in the physical Einstein frame we find scalar field effective potential with a flat region where the confinement dynamics disappears, while in other regions it remains intact. The effective gauge couplings as well as the induced cosmological constant become dynamical. In particular, a conventional Maxwell kinetic term for the gauge field is dynamically generated even if absent in the original theory. We find few interesting classes of explicit solutions: (i) asymptotically (anti-)de Sitter black holes of non-standard type with additional confining vacuum electric potential even for the electrically neutral ones; (ii) non-standard Reissner–Nordström black holes with additional constant vacuum electric field and having non-flat-space–time "hedgehog" asymptotics; (iii) generalized Levi-Civita–Bertotti–Robinson "tube-like" space–times.

1. Introduction

In his analysis in Ref. [1] 't Hooft has shown that in any effective quantum gauge theory, which is able to describe linear confinement phenomena, the energy density of electrostatic field configurations should be a linear function of the electric displacement field in the infrared region. In particular, 't Hooft has developed a consistent quantum approach in which the electric displacement field appears as an "infrared counterterm" (see especially Eq. (5.10) in second item in Ref. [1]).

The simplest way to realize these ideas in flat space–time was proposed in Ref. [2] by considering the following special nonlinear gauge theory:

$$S = \int d^4x L(F^2), \qquad L(F^2) = -\frac{1}{4}F^2 - \frac{f_0}{2}\sqrt{-F^2},$$

$$F^2 \equiv F_{\mu\nu}F^{\mu\nu}, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{1}$$

It has been shown in the first three items in Ref. [2] that the square root of the Maxwell term naturally arises as a result of spontaneous breakdown of scale symmetry of the original scale-invariant Maxwell action with f_0 appearing as an integration constant responsible for the latter spontaneous breakdown. For static field configurations the model (1) yields an electric displacement field $\vec{D} = \vec{E} - \frac{f_0}{\sqrt{2}}\frac{\vec{E}}{|\vec{E}|}$ and the corresponding energy density turns out to be $\frac{1}{2}\vec{E}^2 = \frac{1}{2}|\vec{D}|^2 + \frac{f_0}{\sqrt{2}}|\vec{D}| + \frac{1}{4}f_0^2$, so that it indeed contains a term linear w.r.t. $|\vec{D}|$. The model (1) produces, when coupled to quantized fermions, a confining effective potential $V(r) = -\frac{\beta}{r} + \gamma r$ (Coulomb plus linear one with $\gamma \sim f_0$, see first item in Ref. [2]) which is of the form of the well-known "Cornell" potential [3] in the phenomenological description of quarkonium systems.

To this end it is essential to stress that the Lagrangian $L(F^2)$ (1) contains both the usual Maxwell term as well as the non-analytic square-root function of F^2 and thus it is a *non-standard* form of nonlinear electrodynamics. It is significantly different from the original "square root" Lagrangian $-\frac{f}{2}\sqrt{F^2}$ first proposed by Nielsen and Olesen [4] to describe string dynamics. Also it is important that the square root term in (1) is in the "electrically" dominated form ($\sqrt{-F^2}$) unlike the "magnetically" dominated Nielsen–Olesen form ($\sqrt{F^2}$).

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Let us remark that one could start with the non-Abelian version of the action (1). Since we will be interested in static spherically symmetric solutions, the non-Abelian theory effectively reduces to an Abelian one as pointed out in the first item in Ref. [2].

Coupling of the nonlinear gauge field system (1) to ordinary Einstein gravity was recently studied in [5] where the following interesting new features of the pertinent static spherically symmetric solutions have been found:

(i) Appearance of a constant radial vacuum electric field (in addition to the Coulomb one) in charged black holes within Reissner–Nordström–(anti-)de Sitter space–times, in particular, in electrically neutral black holes with Schwarzschild–(anti-)de Sitter geometry.

(ii) Novel mechanism of *dynamical generation* of cosmological constant through the nonlinear gauge field dynamics of the "square-root" gauge field term: $\Lambda_{\rm eff} = \Lambda_0 + 2\pi f_0^2$ with Λ_0 being the bare cosmological constant.

(iii) In case of vanishing effective cosmological constant ($\Lambda_0 < 0$, $|\Lambda_0| = 2\pi f_0^2$) the resulting Reissner–Nordström-type black hole, apart from carrying an additional constant vacuum electric field, turns out to be *non-asymptotically flat* – a feature resembling the gravitational effect of a hedgehog [6].

(iv) Appearance of confining-type effective potential in charged test particle dynamics in the above black hole backgrounds (cf. Eq. (43) below).

(v) New "tube-like" solutions of Levi-Civita–Bertotti–Robinson [7] type, i.e., with space–time geometry of the form $\mathcal{M}_2 \times S^2$, where $\mathcal{M}_2$ is a two-dimensional anti-de Sitter, Rindler or de Sitter space depending on the relative strength of the electric field w.r.t. the coupling f_0 of the square-root gauge field term.

Let us also mention the recent paper [8] where coupling of ordinary Einstein gravity to the pure "square-root" gauge field Lagrangian ($L(F^2) = -\frac{f_0}{2}\sqrt{-F^2}$) is discussed. A new interesting feature in this model is the existence of dyonic solutions.

In the present Letter we will consider $f(R)$-gravity[1] with the simplest nonlinear $f(R) = R + \alpha R^2$ coupled to dilaton ϕ and the "square-root" nonlinear gauge system (1). The main purpose is to study possible new features due to the combined effect of the simultaneous presence of both the R^2-gravity term and the confinement-producing "square-root" of the Maxwell term. This will be achieved by starting within the first-order (Palatini) formulation of $f(R)$-gravity (e.g. [9,11]) and systematically deriving the effective action of the full theory in the physical "Einstein" frame. The resulting Einstein-frame action is of the form of standard Einstein gravity interacting with a dilaton ϕ and the special nonlinear gauge system with the square-root term (1), where all couplings become dynamically dependent on ϕ. The latter include the effective strength of the standard Maxwell term, the effective coupling constant of the square-root gauge field term and the effective cosmological constant. This, in particular, means:

(1) If we start with *no* standard Maxwell kinetic term in the original theory, a nontrivial Maxwell Lagrangian term will nevertheless be dynamically generated with a ϕ-dependent strength in the "Einstein" frame. The same is true about the dynamically generated cosmological constant out of a zero bare one.

(2) For certain regions of values for the constant dilaton ϕ the effective coupling constant $f_{\rm eff}(\phi)$ of the "square-root" gauge field term will be vanishing indicating confinement/deconfinement transition (let us recall that the coupling constant f_0 in (1) measures the strength of the effective linear confining potential, see first item in Ref. [2] and Eq. (43) below). The above mentioned regions also correspond to flat regions of the effective scalar potential. The latter could be used as regions in field space where an inflationary phase for the universe took place according to the requirements in the "new inflationary" scenarios [12].

(3) Furthermore, we find generalizations of the black hole and "tube-like" space–time solutions mentioned in (v) above where now their parameters are ϕ-dependent.

We particularly stress that both effects (1)–(2) are entirely due to both parameters α (of R^2-gravity) and f_0 (of nonlinear "square-root" gauge theory) simultaneously being non-zero.

Let us also mention a recent work [13] where an R^2-gravity interacting with Born–Infeld nonlinear electrodynamics has been studied and new types of black hole solutions with different structure of the horizons and singularities have been found.

2. Derivation of Einstein-frame effective action of R^2-gravity-matter system

The action describing the coupling of $f(R) = R + \alpha R^2$ gravity (possibly with a bare cosmological constant Λ_0) to a dilaton ϕ and the nonlinear gauge field system with a square-root of the Maxwell term (1) known to produce QCD-like confinement in flat space–time [2] is given by:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\bigl(f\bigl(R(g,\Gamma)\bigr) - 2\Lambda_0\bigr) + L\bigl(F^2(g)\bigr) + L_D(\phi, g)\right], \tag{2}$$

$$f\bigl(R(g,\Gamma)\bigr) = R(g,\Gamma) + \alpha R^2(g,\Gamma),$$
$$R(g,\Gamma) = R_{\mu\nu}(\Gamma)g^{\mu\nu}, \tag{3}$$

$$L\bigl(F^2(g)\bigr) = -\frac{1}{4e^2}F^2(g) - \frac{f_0}{2}\sqrt{\varepsilon F^2(g)}, \tag{4}$$

$$F^2(g) \equiv F_{\kappa\lambda}F_{\mu\nu}g^{\kappa\mu}g^{\lambda\nu}, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{5}$$

$$L_D(\phi, g) = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi). \tag{6}$$

In (2)–(3) $R_{\mu\nu}(\Gamma)$ indicates the Ricci curvature in the first order (Palatini) formalism, i.e., the space–time metric $g_{\mu\nu}$ and the affine connection $\Gamma^\mu_{\nu\lambda}$ are a priori independent variables. Further notations indicate: $g \equiv \det\|g_{\mu\nu}\|$; the sign factor $\varepsilon = \pm 1$ in the square root term in (4) corresponds to "magnetic" or "electric" dominance; f_0 is a positive coupling constant.

It is important to stress that:

(i) We will consider in what follows constant dilaton field, i.e., ignoring the kinetic term in (6).

(ii) We will be particularly interested in the special cases $\Lambda_0 = 0$ (*no* bare cosmological constant term) and/or $e^2 \to \infty$ in (4), i.e., a nonlinear gauge field action (4) *without* a bare Maxwell term ($L(F^2(g)) = -\frac{f_0}{2}\sqrt{\varepsilon F^2(g)}$).

As we will show below, both a standard kinetic Maxwell term for the gauge field with a variable ϕ-dependent strength as well as a ϕ-dependent cosmological constant are *dynamically generated* as a combined effect of the R^2-gravity term and the nonlinear "square-root" Maxwell term in (4).

The equations of motion resulting from the action (2) read:

$$R_{\mu\nu}(\Gamma) = \frac{1}{f'_R}\left[\kappa^2 T_{\mu\nu} + \frac{1}{2}f\bigl(R(g,\Gamma)\bigr)g_{\mu\nu}\right],$$
$$f'_R \equiv \frac{df(R)}{dR} = 1 + 2\alpha R(g,\Gamma), \tag{7}$$

$$\nabla_\lambda\bigl(\sqrt{-g}f'_R g^{\mu\nu}\bigr) = 0, \tag{8}$$

[1] For a recent review of $f(R)$-gravity see e.g. [9] and references therein. The first R^2-model (within the second order formalism), which was also the first inflationary model, was proposed in [10].

$$\partial_\nu\left(\sqrt{-g}\left[1/e^2+\varepsilon\frac{f_0}{\sqrt{\varepsilon F^2(g)}}\right]F_{\kappa\lambda}g^{\mu\kappa}g^{\nu\lambda}\right)=0. \tag{9}$$

About the dilaton, see Eq. (32) below. Here the total energy–momentum tensor is given by:

$$T_{\mu\nu}=\left[L\big(F^2(g)\big)+L_D(\phi,g)-\frac{1}{\kappa^2}\Lambda_0\right]g_{\mu\nu} + \left(1/e^2+\frac{\varepsilon f_0}{\sqrt{\varepsilon F^2(g)}}\right)F_{\mu\kappa}F_{\nu\lambda}g^{\kappa\lambda}+\partial_\mu\phi\partial_\nu\phi \tag{10}$$

with $L(F^2(g))$ and $L_D(\phi)$ as in (4)–(6).

Taking the trace of (7) and using the explicit form of $f(R)$ in (2) one gets for the Ricci scalar curvature:

$$R(g,\Gamma)=-\kappa^2T(g),\qquad T(g)=T_{\mu\nu}g^{\mu\nu}. \tag{11}$$

It can easily be shown (cf. [14]) that Eq. (8) leads to the relation $\nabla_\lambda(f'_R g_{\mu\nu})=0$ and thus it implies transition to the "physical" Einstein-frame metrics $h_{\mu\nu}$ via conformal rescaling of the original metric $g_{\mu\nu}$:

$$g_{\mu\nu}=\frac{1}{f'_R}h_{\mu\nu},\qquad \Gamma^\mu_{\nu\lambda}=\frac{1}{2}h^{\mu\kappa}(\partial_\nu h_{\lambda\kappa}+\partial_\lambda h_{\nu\kappa}-\partial_\kappa h_{\nu\lambda}). \tag{12}$$

Using (12) and taking into account (11) one can rewrite gravity Eqs. (7) within the Einstein frame as follows (from now on all space–time indices are raised/lowered by $h_{\mu\nu}$):

$$R^\mu_\nu(h)=\kappa^2\left[\frac{1}{f'_R}T^\mu_\nu(h)-\frac{1}{4}\left(1+\frac{1}{f'_R}\right)T(h)\delta^\mu_\nu\right], \tag{13}$$

where:

$$T_{\mu\nu}(h)=\left[-\frac{1}{f'_R}\big(V(\phi)+\Lambda_0/\kappa^2\big)+\left(-\frac{1}{2}f_0\sqrt{\varepsilon F^2(h)}-X(\phi,h)\right) - f'_R\frac{1}{4e^2}F^2(h)\right]h_{\mu\nu}+\left[\frac{f'_R}{e^2}+\frac{\varepsilon f_0}{\sqrt{\varepsilon F^2(h)}}\right]F_{\mu\kappa}F_{\nu\lambda}h^{\kappa\lambda} + \partial_\mu\phi\partial_\nu\phi, \tag{14}$$

$$T(h)=T_{\mu\nu}(h)h^{\mu\nu}=-f_0\sqrt{\varepsilon F^2(h)}+2X(\phi,h) - 4\frac{1}{f'_R}\big(V(\phi)+\Lambda_0/\kappa^2\big), \tag{15}$$

with short-hand notations:

$$F^2(h)\equiv F_{\kappa\lambda}F_{\mu\nu}h^{\kappa\mu}h^{\lambda\nu},\qquad X(\phi,h)\equiv-\frac{1}{2}h^{\mu\nu}\partial_\mu\phi\partial_\nu\phi, \tag{16}$$

and

$$\begin{aligned} f'_R&=\big[1+2\alpha\kappa^2T(h)\big]^{-1}\\ &=\big[1+8\alpha\big(\kappa^2V(\phi)+\Lambda_0\big)\big]\\ &\quad\times\big[1-2\alpha\kappa^2\big(f_0\sqrt{\varepsilon F^2(h)}+h^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\big)\big]^{-1}. \end{aligned} \tag{17}$$

Accordingly, using (17) the nonlinear gauge field Eqs. (9) become:

$$\partial_\nu\left(\sqrt{-h}\left[\frac{1}{e^2_{\rm eff}(\phi)}+\frac{\varepsilon f_{\rm eff}(\phi)}{\sqrt{\varepsilon F^2(h)}}\right]F_{\kappa\lambda}h^{\mu\kappa}h^{\nu\lambda}\right)=0, \tag{18}$$

where we introduced the dynamical couplings:

$$\frac{1}{e^2_{\rm eff}(\phi)}=\frac{1}{e^2}-\frac{2\varepsilon\alpha\kappa^2f_0^2}{1+8\alpha(\kappa^2V(\phi)+\Lambda_0)}, \tag{19}$$

$$f_{\rm eff}(\phi)=f_0\frac{1+4\alpha\kappa^2X(\phi,h)}{1+8\alpha(\kappa^2V(\phi)+\Lambda_0)}. \tag{20}$$

Now, as an important step using the explicit expressions (14)–(17) we can cast Eqs. (13) in the form of *standard* Einstein equations:

$$R^\mu_\nu(h)=\kappa^2\left(T_{\rm eff}{}^\mu_\nu(h)-\frac{1}{2}\delta^\mu_\nu T_{\rm eff}{}^\lambda_\lambda(h)\right) \tag{21}$$

with energy–momentum tensor of the following form:

$$T_{{\rm eff}\,\mu\nu}(h)=h_{\mu\nu}L_{\rm eff}(h)-2\frac{\partial L_{\rm eff}}{\partial h^{\mu\nu}} \tag{22}$$

where (using notations (19)–(20)):

$$L_{\rm eff}(h)=-\frac{1}{4e^2_{\rm eff}(\phi)}F^2(h)-\frac{1}{2}f_{\rm eff}(\phi)\sqrt{\varepsilon F^2(h)} + \frac{X(\phi,h)(1+2\alpha\kappa^2X(\phi,h))-V(\phi)-\Lambda_0/k^2}{1+8\alpha(\kappa^2V(\phi)+\Lambda_0)}. \tag{23}$$

Thus, all equations of motion of the original $f(R)$-gravity/nonlinear-gauge-field system (2)–(6) with metric $g_{\mu\nu}$ can be equivalently derived from the following Einstein/nonlinear-gauge-field/dilaton action:

$$S_{\rm eff}=\int d^4x\sqrt{-h}\left[\frac{R(h)}{2\kappa^2}+L_{\rm eff}(h)\right], \tag{24}$$

where $R(h)$ is the standard Ricci scalar of the metric $h_{\mu\nu}$ and $L_{\rm eff}(h)$ is as in (23).

Let us particularly stress that in the absence of ordinary kinetic Maxwell term in the original system ($e^2\to\infty$ in (4)), such term is nevertheless *dynamically generated* in the Einstein-frame action (23)–(24):

$$S_{\rm maxwell}=-\frac{1}{2}\alpha\kappa^2f_0^2\int d^4x\sqrt{-h}\frac{F_{\kappa\lambda}F_{\mu\nu}h^{\kappa\mu}h^{\lambda\nu}}{1+8\alpha(\kappa^2V(\phi)+\Lambda_0)}, \tag{25}$$

hence we will assume $\alpha>0$. The Maxwell term generation occurs from the R^2 term in the original theory in the process of passing to the Einstein frame due to the on-shell relation (11) where a *non-zero* trace of the energy–momentum tensor is produced by the nonlinear "square-root" gauge field term: $T(g)=-f_0\sqrt{-F^2(g)}+$ (ϕ-contribution).

3. Non-Standard Black Hole Solutions

In what follows we will consider the case of constant dilaton ϕ extremizing the effective Lagrangian (23), i.e., we will ignore the kinetic $X(\phi,h)$-terms in (23), and from now on we will concentrate on the "electric-dominant" case $\varepsilon=-1$:

$$L_{\rm eff}=-\frac{1}{4e^2_{\rm eff}(\phi)}F^2(h)-\frac{1}{2}f_{\rm eff}(\phi)\sqrt{-F^2(h)}-V_{\rm eff}(\phi). \tag{26}$$

Here the effective scalar potential and the effective couplings in (26) are explicitly given by:

$$V_{\rm eff}(\phi)=\frac{V(\phi)+\frac{\Lambda_0}{\kappa^2}}{1+8\alpha(\kappa^2V(\phi)+\Lambda_0)}, \tag{27}$$

$$\frac{1}{e^2_{\rm eff}(\phi)}=\frac{1}{e^2}+\frac{2\alpha\kappa^2f_0^2}{1+8\alpha(\kappa^2V(\phi)+\Lambda_0)}, \tag{28}$$

$$f_{\rm eff}(\phi)=\frac{f_0}{1+8\alpha(\kappa^2V(\phi)+\Lambda_0)}. \tag{29}$$

Since both f_0 and $f_{\rm eff}(\phi)$ (couplings of the "square-root" gauge field terms) must be positive (they determine the strength of the linear confining part in the effective "Cornell" potential, cf. first item in Ref. [2] and (43) below) we must have:

$$1 + 8\alpha\big(\kappa^2 V(\phi) + \Lambda_0\big) > 0. \tag{30}$$

An important property of (26) is that the derivatives w.r.t. ϕ of the dynamical couplings (28)–(29) are both extremized simultaneously with the extremization of the effective scalar potential (27):

$$\frac{\partial f_{\mathrm{eff}}}{\partial\phi} = -8\alpha\kappa^2 f_0 \frac{\partial V_{\mathrm{eff}}}{\partial\phi},$$
$$\frac{\partial}{\partial\phi}\left(\frac{1}{e^2_{\mathrm{eff}}(\phi)}\right) = -16\alpha^2\kappa^4 f_0^2 \frac{\partial V_{\mathrm{eff}}}{\partial\phi} \to \frac{\partial L_{\mathrm{eff}}}{\partial\phi} \sim \frac{\partial V_{\mathrm{eff}}}{\partial\phi}. \tag{31}$$

Therefore at the extremum of L_{eff} (26) ϕ must satisfy:

$$\frac{\partial V_{\mathrm{eff}}}{\partial\phi} = \frac{V'(\phi)}{[1+8\alpha(\kappa^2 V(\phi)+\Lambda_0)]^2} = 0. \tag{32}$$

There are two generic cases:

(a) Eq. (32) is satisfied for some finite-value ϕ_0 being an extremum of the original potential $V(\phi)$:

$$V'(\phi_0) = 0. \tag{33}$$

(b) For polynomial or exponentially growing original $V(\phi)$, so that $V(\phi) \to \infty$ when $\phi \to \infty$, we have:

$$\frac{\partial V_{\mathrm{eff}}}{\partial\phi} \to 0, \qquad V_{\mathrm{eff}}(\phi) \to \frac{1}{8\alpha\kappa^2} = \mathrm{const} \quad \text{when } \phi \to \infty, \tag{34}$$

i.e., for sufficiently large values of ϕ we find a "flat region" in the effective scalar potential (27). Also, in this case we have instead of (28)–(29):

$$f_{\mathrm{eff}} \to 0, \qquad e^2_{\mathrm{eff}} \to e^2 \tag{35}$$

and (26) reduces to:

$$L^{(0)}_{\mathrm{eff}} = -\frac{1}{4e^2}F^2(h) - \frac{1}{8\alpha\kappa^2}. \tag{36}$$

Now, the action (24) with the matter Lagrangian $L_{\mathrm{eff}}(h)$ as in (26) is of the same general form as the action of the model describing ordinary Einstein gravity interacting with the nonlinear gauge field theory containing square root of the Maxwell term, which was discussed in Ref. [5], with the only difference being the substitutions of the ordinary parameters e^2, f_0, Λ_0 with the effective ϕ-dependent ones from (27)–(29), where the scalar field ϕ is constant. Therefore, we can implement the same steps as in [5] to find static spherically symmetric solutions of the system (24), which is the effective Einstein-frame form of the original $f(R) = R + \alpha R^2$-gravity/nonlinear-gauge-field theory (2).

The static radial electric field F_{0r} contains both Coulomb ($\sim \frac{1}{r^2}$) as well as a *constant non-zero vacuum* piece (here and below $\phi = \mathrm{const}$):

$$|F_{0r}| = \left(\frac{1}{e^2} + \frac{2\alpha\kappa^2 f_0^2}{1+8\alpha(\kappa^2 V(\phi)+\Lambda_0)}\right)^{-1} \times \frac{f_0/\sqrt{2}}{1+8\alpha(\kappa^2 V(\phi)+\Lambda_0)} + \frac{|Q|}{\sqrt{4\pi}r^2}. \tag{37}$$

The solution for the Einstein-frame metric $h_{\mu\nu}$ reads:

$$ds_h^2 = -A(r)dt^2 + \frac{dr^2}{A(r)} + r^2\big(d\theta^2 + \sin^2\theta d\varphi^2\big), \tag{38}$$

where:

$$A(r) = 1 - \frac{\kappa^2|Q|f_0}{\sqrt{8\pi}[1+8\alpha(\kappa^2 V(\phi)+\Lambda_0)]} - \frac{2m\kappa^2}{8\pi r} + \left[\frac{1}{e^2} + \frac{2\alpha\kappa^2 f_0^2}{1+8\alpha(\kappa^2 V(\phi)+\Lambda_0)}\right]\frac{\kappa^2 Q^2}{8\pi r^2} - \frac{\Lambda_{\mathrm{eff}}(\phi)}{3}r^2, \tag{39}$$

with a total dynamical effective cosmological constant:

$$\Lambda_{\mathrm{eff}}(\phi) = \frac{\Lambda_0 + \kappa^2 V(\phi) + \kappa^2 e^2 f_0^2/4}{1+8\alpha(\Lambda_0 + \kappa^2 V(\phi) + \kappa^2 e^2 f_0^2/4)} \tag{40}$$

and with *a priori* free mass parameter m.

Therefore, in the case of ordinary extremum of the effective scalar potential (32)–(33) the properties of the solution depend on the sign of the expression $\Lambda_0 + \kappa^2 V(\phi_0) + \kappa^2 e^2 f_0^2/4$, which determines the sign of $\Lambda_{\mathrm{eff}}(\phi_0)$. We find:

(i) For positive/negative values of $\Lambda_0 + \kappa^2 V(\phi_0) + \kappa^2 e^2 f_0^2/4$, so that $\Lambda_{\mathrm{eff}}(\phi_0) > 0$ ($\Lambda_{\mathrm{eff}}(\phi_0) < 0$), the solution (37)–(40) describes Reissner–Nordström–(anti-)de Sitter-type black hole carrying an additional vacuum radial electric field with magnitude:

$$|E_{\mathrm{vac}}| = \left(\frac{1}{e^2} + \frac{2\alpha\kappa^2 f_0^2}{1+8\alpha(\kappa^2 V(\phi_0)+\Lambda_0)}\right)^{-1} \frac{f_0/\sqrt{2}}{1+8\alpha(\kappa^2 V(\phi_0)+\Lambda_0)}. \tag{41}$$

(ii) In the special case $\Lambda_0 + \kappa^2 V(\phi_0) + \kappa^2 e^2 f_0^2/4 = 0$ when $\Lambda_{\mathrm{eff}}(\phi_0) = 0$, the solution (37)–(40) yields a Reissner–Nordström-type non-standard black hole which, apart from carrying the additional vacuum radial electric field (41), exhibits *non-flat* "hedgehog"-type space–time asymptotics [6]:

$$A(r)|_{r\to\infty} \simeq 1 - \frac{\kappa^2|Q|f_0}{\sqrt{8\pi}[1+8\alpha(\kappa^2 V(\phi_0)+\Lambda_0)]} < 1. \tag{42}$$

(iii) In the case of anti-de Sitter or hedgehog-type asymptotics the condition $\Lambda_0 + \kappa^2 V(\phi_0) + \kappa^2 e^2 f_0^2/4 \leqslant 0$ together with (30) implies an upper bound for α: $\alpha < (2\kappa^2 e^2 f_0^2)^{-1}$.

(iv) Following the same steps as in Section 4 of first item in Ref. [5] we find a linear (w.r.t. r) confining piece in the effective potential of test charged particle dynamics in the above black hole backgrounds:

$$\frac{\sqrt{2}\mathcal{E}|q_0|}{m_0^2} e_{\mathrm{eff}}(\phi_0) f_{\mathrm{eff}}(\phi_0) r, \tag{43}$$

where $\mathcal{E}, m_0, q_0$ are energy, mass and charge of the test particle and the effective gauge field couplings are as in (28)–(29).

On the other hand, in the case of "flat region" for the effective scalar potential (34)–(35) the solution (37)–(40) reduces to an *ordinary* Reissner–Nordström–de Sitter black hole:

$$|F_{0r}| = \frac{|Q|}{\sqrt{4\pi}r^2}, \qquad A(r) = 1 - \frac{2m}{r} + \frac{\kappa^2 Q^2}{8\pi r^2} - \frac{1}{24\alpha}r^2, \tag{44}$$

with an *induced* cosmological constant $\Lambda_{\mathrm{eff}} = 1/8\alpha$, which is *completely independent* of the bare cosmological constant Λ_0. Moreover, in the regime (35) the linear confining potential for the test particles (43) disappears.

4. Generalized Levi-Civita–Bertotti–Robinson solutions

Following the same steps as in second and third items in Ref. [5] we can find explicit static solutions of generalized Levi-Civita–Bertotti–Robinson (LCBR) type [7] of the system (24)–(26).

For definiteness we will concentrate on the case of "electric dominance" and for simplicity we will use units with Newton constant $G_N = 1$, i.e., $\kappa^2 = 8\pi$, and $e^2 = 1$. These generalized LCBR-type space–times are "tube-like" solutions with space–time geometry of the form $\mathcal{M}_2 \times S^2$ where $\mathcal{M}_2$ is some two-dimensional manifold with coordinates (t, η):

$$ds_h^2 = -A(\eta)dt^2 + \frac{d\eta^2}{A(\eta)} + r_0^2\left(d\theta^2 + \sin^2\theta d\phi^2\right),$$
$$-\infty < \eta < \infty, \; r_0 = \text{const.} \tag{45}$$

and with a constant radial static electric field:

$$F_{0\eta} = c_F = \text{arbitrary const.} \tag{46}$$

The Einstein equations corresponding to (24)–(26) yield the following relation for r_0:

$$\frac{1}{r_0^2} = \frac{4\pi}{1+8\alpha\Lambda(\phi_0)}\left[\left(1+8\alpha\left(\Lambda(\phi_0)+2\pi f_0^2\right)\right)c_F^2 + \frac{1}{4\pi}\Lambda(\phi_0)\right] \tag{47}$$

with the short-hand notation $\Lambda(\phi_0) \equiv 8\pi V(\phi_0) + \Lambda_0$, and the following simple differential equation for the metric coefficient $A(\eta)$ in (45):

$$\partial_\eta^2 A(\eta) = \frac{8\pi}{1+8\alpha\Lambda(\phi_0)} K(c_F), \tag{48}$$

$$K(c_F) \equiv \left(1+8\alpha\left(\Lambda(\phi_0)+2\pi f_0^2\right)\right)c_F^2 - \sqrt{2} f_0 |c_F| - \frac{1}{4\pi}\Lambda(\phi_0). \tag{49}$$

In the case of "flat region" of the effective scalar potential (34) $\Lambda(\phi_0) \to \infty$, so that Eqs. (47)–(48) simplify:

$$\frac{1}{r_0^2} = 4\pi c_F^2 + \frac{1}{8\alpha}, \qquad \partial_\eta^2 A(\eta) = 8\pi c_F^2 - \frac{1}{4\alpha}. \tag{50}$$

As in last item in Ref. [5], there are three distinct types of generalized LCBR solutions depending on the sign of the factor $K(c_F)$ in (48)–(49).

(A) $AdS_2 \times S^2$ with strong constant vacuum electric field $F_{0\eta} = c_F$, where AdS_2 is two-dimensional anti-de Sitter space with:

$$A(\eta) = 4\pi K(c_F)\eta^2, \qquad K(c_F) > 0, \tag{51}$$

in the metric (45), η being the Poincare patch space-like coordinate. The magnitude $|c_F|$ of the vacuum electric field must satisfy the inequalities:

$$|c_F| > \frac{f_0}{\sqrt{2}[1+8\alpha(\Lambda(\phi_0)+2\pi f_0^2)]} \times \left[1+\sqrt{1+\frac{\Lambda(\phi_0)}{2\pi f_0^2}\left[1+8\alpha\left(\Lambda(\phi_0)+2\pi f_0^2\right)\right]}\right] \tag{52}$$

for $\Lambda(\phi_0) > \max\{-2\pi f_0^2, -\frac{1}{8\alpha}\}$;

$$c_F^2 > \frac{|\Lambda(\phi_0)|}{4\pi[1+8\alpha(\Lambda(\phi_0)+2\pi f_0^2)]},$$
$$\text{for } -\frac{1}{8\alpha} < \Lambda(\phi_0) < -2\pi f_0^2. \tag{53}$$

In the "flat region" case (50) $|c_F| > (32\pi\alpha)^{-\frac{1}{2}}$.

(B) $Rind_2 \times S^2$ with constant vacuum electric field $F_{0\eta} = c_F$ when the factor $K(c_F) = 0$. Here $Rind_2$ is the flat two-dimensional Rindler space with:

$$A(\eta) = \eta \quad \text{for } 0 < \eta < \infty \quad \text{or}$$
$$A(\eta) = -\eta \quad \text{for } -\infty < \eta < 0 \tag{54}$$

in the metric (45) and:

$$|c_F| = \frac{f_0}{\sqrt{2}[1+8\alpha(\Lambda(\phi_0)+2\pi f_0^2)]} \times \left[1+\sqrt{1+\frac{\Lambda(\phi_0)}{2\pi f_0^2}\left[1+8\alpha\left(\Lambda(\phi_0)+2\pi f_0^2\right)\right]}\right] \tag{55}$$

for $\Lambda(\phi_0) > \max\{-2\pi f_0^2, -\frac{1}{8\alpha}\}$. In the "flat region" case (50) $|c_F| = (32\pi\alpha)^{-\frac{1}{2}}$.

(C) $dS_2 \times S^2$ with weak constant vacuum electric field $F_{0\eta} = c_F$, where dS_2 is two-dimensional de Sitter space with:

$$A(\eta) = 1 - 4\pi|K(c_F)|\eta^2, \qquad K(c_F) < 0, \tag{56}$$

in the metric (45). The magnitude $|c_F|$ of the vacuum electric field must satisfy:

$$|c_F| < \frac{f_0}{\sqrt{2}[1+8\alpha(\Lambda(\phi_0)+2\pi f_0^2)]} \times \left[1+\sqrt{1+\frac{\Lambda(\phi_0)}{2\pi f_0^2}\left[1+8\alpha\left(\Lambda(\phi_0)+2\pi f_0^2\right)\right]}\right], \tag{57}$$

where $\Lambda(\phi_0) > \max\{-2\pi f_0^2, -\frac{1}{8\alpha}\}$. In the "flat region" case (50) $|c_F| < (32\pi\alpha)^{-\frac{1}{2}}$.

5. Discussion

In the present Letter we have considered $f(R) = R + \alpha R^2$-gravity within the first-order (Palatini) formalism coupled to dilaton and a special kind of nonlinear gauge field system containing a square-root of the standard Maxwell term, which is known to produce a QCD-like confinement. We have derived the explicit form of the dynamically equivalent "physical" Einstein-frame effective theory displaying the following significant properties:

(i) The effective gauge field couplings as well as the effective cosmological constant become functions of the constant dilaton in such a way that even in the event of absence of kinetic Maxwell term for the gauge field and/or absence of bare cosmological constant in the original theory, the latter are nevertheless dynamically generated in the Einstein-frame effective theory.

(ii) There are two interesting regimes for the constant dilaton ϕ: (a) either as a minimum of the bare scalar potential at some finite value ϕ_0, which coincides with the minimum ϕ_0 of the effective scalar potential in the Einstein-frame theory, or (b) ϕ belongs to a "flat region" of the effective scalar potential, which corresponds to a fast growing at infinity bare scalar potential. In the first case the effective coupling of the confinement-producing "square-root" gauge field term remains finite, whereas in the second case it vanishes and so does the confining feature. This picture resembles the "MIT bag" structure [15] where inside the "bag" a regular gauge-field dynamics holds, while outside the "bag" the gauge fields are confined. Moreover, the effective dilaton potential with a "flat region" can be used for inflation.

(iii) The effective coupling constants in the Einstein-frame theory satisfy the "least coupling principle" of Damour and Polyakov [16], namely, any extremal point ϕ_0 of the scalar effective potential (as function of the dilaton) is simultaneously an extremal point of the effective gauge couplings. This property is crucial for the consistency of the solutions here obtained.

(iv) We derived new solutions describing non-standard black holes and Levi-Civita–Bertotti–Robinson-type "tube-like" space–times, generalizing those found in [5], in that now the pertinent constant vacuum electric fields and the non-flat "hedgehog" space–time asymptotics depend on the dilaton value ϕ_0.

Let us also point out that an R^2-gravity theory coupled to non-linear gauge field system with Maxwell and "square-root" terms and a dilaton was earlier studied in the context of the so called two-measure gravity models in Ref. [17]. The results there about the explicit form of the Einstein-frame effective theory resembles those obtained in the present Letter.

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